SENSOR TIP

Understanding Linearity

Understanding linearity

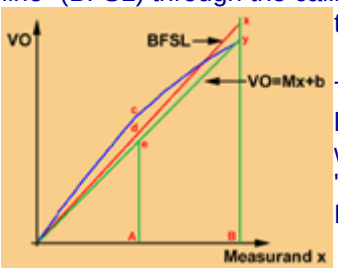
There are several ways to interpret and represent linearity of a transducer. Those most commonly used are... End point linearity, Best fit straight line (BFSL), and Least squares BFSL linearity.

End Point Linearity

From the graph plotted of Voltage Output against Increase in Measurand which usually appears as a curve, a straight line is drawn from the zero point to the full scale output point. Usually the point which deviates most from the simple straight line will be used to specify the 'linearity' of the transducer. This is quoted as a percentage of the normal full scale output of the transducer.

Best fit straight line method of definition

In practice, the relationship between the measurand and the output of most transducers is not perfectly linear and it is therefore necessary to find a way of using these devices to achieve the most accurate overall results. This can be achieved by constructing what is known as the "Best fit straight line" (BFSL) through the calibration points in such a way that the maximum deviation of the curve from the line is minimised as indicated in the figure below.



This diagram simply illustrates how, instead of drawing a straight line between the origin and the end point at "B", we can construct a line which effectively halves the maximum deviation that would appear at point "A" and shares it with the end point at "B". Thus the deviation from the BFSL, and hence the linearity error, is effectively halved.

However, it should be appreciated that this is simply a means of interpretation of the results and relies entirely on the system, including the measuring equipment, being set up to take advantage of this technique.

To take advantage of this method in the case illustrated, it Is necessary to set up the system so that when the transducer is at full scale 'y' at point 'B' the indicated output would be set to a value of 'x'. This then effectively halves the error indicated at 'A' which would now deviate from the BFSL by the amount rather than if we had simply constructed an 'end point straight line'.

It can be clearly seen that this enables us to effectively halve the errors in the system and thus record better results over the whole measuring range. Note however that although it is quite easy to construct a BFSL on a graph of transducer output against measurand, it is not so easy to do when faced with a simple set of figures. In practice we need a better, quicker and more consistent means of determining the BFSL which can also take into account transducers with bi-polar outputs such as LVDTs and tension/compression load cells.

The answer to this problem is the 'Least squares method' of determining the BFSL. This is a statistical method, which enables the BFSL to be determined mathematically over any chosen working range, and is the most suitable method for use in computerised calibration systems.

Least Squares Best Fit Straight Line Method

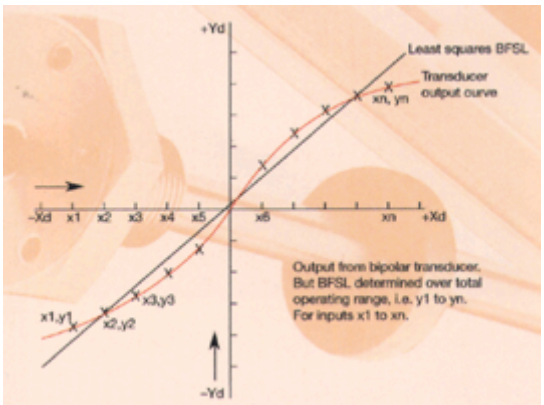
In the previous issues of Transducer Action, we have discussed 'End point linearity' and 'Best fit straight line' methods of definition. Our concluding article deals with the 'Least Squares Best Fit Straight Line' method, preferred by most transducer manufacturers because it provides the closest possible best fit to all data points on the curve, and can be most readily adapted to the computerised calibration systems in common use.

The Least Squares Best Fit Straight Line is a statistical method and as such may not be a 'purist' approach but provided the characteristics of the transducers are correctly optimised at the design and development stage and are represented by a continuous smooth curve, the assessment is meaningful and accurate.

In practice, up to 20 calibration points will be taken over the whole working range of the transducer and the measured input and output values at each point used to provide the data for calculation of the slope of the 'Least Squares Best Fit Straight Line' using the following equation:

$$\text{Slope(s)} = \frac{\sum_{d=1}^{n} Xd\ Yd}{\sum_{d=1}^{n} Xd^2}$$

Where: Xd = known input data points
Yd = actual sensor output at each Xd data point
n = number of data points



Having mathematically determined the slope of the best fit straight line it is then possible to determine the maximum deviation of any point from this line using the equation:

$$\%\ d = 100\left(\frac{Yd_{max} - SXd}{SX_{fr}}\right)$$

where fr = full working range

As with all other methods the maximum deviation value would be expressed as a percentage of the total linear range of the device. It should be noted that since this evaluation is done over the total range of the transducer the best fit straight line may not pass through the zero point of a bipolar device such as an LVDT, Universal load cell, differential pressure transducer etc. However, this does not have any great practical significance since the zero output point is usually adjustable electronically anyway.

In general it can be readily agreed that this method is by far the most efficient one, but it must be used with care and understanding. Clearly because of its statistical nature, the number of data points taken will have a direct bearing on the ultimate validity of the assessment. In practice, the more uneven the characteristic curve, the more data points must be used in order to take the incremental non-linearity into account.